REPLY TO COMMENTS OF BASSI, GHIRARDI, AND TUMULKA ON THE FREE WILL THEOREM

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0. INTRODUCTION

In [1], Bassi and Ghirardi state that, contrary to our assertion in [2], the Free Will Theorem does not show the impossibility of a relativistic GRW theory. In this paper we argue that [1] is in error on three points.

1. The first and most crucial one is their claim that one of our axioms is false. In [2] we deduced from three physical axioms SPIN, TWIN and FIN[1] and the assumption FREE of experimental free will, that spin 1 particle responses are also free. Bassi and Ghirardi do not take issue with our proof, but claim that

> "after the work of Bell, it is well known that the conclusion is a different one, that FIN is in fact false: Nature is non-local, i.e., FIN is wrong ..."

Their claim is based on equating the Bell locality condition ([1] Section 2.2), which Bell showed was false, with FIN:

> "... Bell's definition of locality, which in some sense is the analog of the FIN axiom ..."

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[1]For the reader's convenience, we briefly recall that SPIN is the assertion that if squared spin measurements are made by an experimenter A on a spin 1 particle a in the three directions of an orthogonal frame, the answers will be $1, 0, 1$ in some order. TWIN is the assertion that two such particles a and b can be put into the singlet state and spatially separated, and then if corresponding experimenters A and B measure their squared spins in the same directions, they will obtain the same results. Finally, FIN is the assertion that information cannot be effectively transmitted at more than a fixed finite speed, the speed of light.

We agree that Nature has non-local correlations – indeed TWIN expresses such correlations – and that Bell's locality assumption is false. We argue in Section 1 that nevertheless FIN is true, because it follows from causality and relativity. It is wrong to conflate FIN with Bell locality.

2. The second point is that whereas we deduced from the Free Will Theorem that a relativistic GRW theory is impossible, [1] says that the theory is "Lorentz invariant in the stochastic sense" and argue that this is as invariant as quantum mechanics allows. To some extent this is an argument about the most appropriate meaning of "relativistic." In Section 2 we argue that, despite Bell non-locality, the predictions of quantum mechanics are fully Lorentz invariant, and that this should therefore be the natural meaning. Furthermore, we show that not only do the GRW jumps violate Lorentz invariance as [1] admits, but that GRW theories lead to macroscopic violations of relativity and/or causality and our assumption FREE of experimental freedom.

3. The final point is that [1] describes our notion of information as vague, and that the GRW jumps should not count as information. We argue in Section 3 that we have simply allowed the notion of bits of information to be as general as possible in order to make our theorem as strong as possible, and that excluding jumps from information cannot help since, as we said above, they lead to macroscopic violations of relativity.

1. BELL LOCALITY IS FALSE, BUT FIN IS TRUE

Along with Bassi and Ghirardi, we believe that the predictions of quantum mechanics are true (at least to very high accuracy). Our axioms SPIN and TWIN are among them, as is the failure of Bell locality. We also believe that (special) relativity is true to a similar high accuracy, and that the causality principle, that effects cannot happen at earlier times than their causes, is valid. By change of frame, this implies that cause and effect cannot be space-like separated, which in turn implies FIN, since superluminally transmitted information could cause an effect.

Since the quantum-mechanical predictions SPIN and TWIN that we use are relativistically invariant, it follows that there can be no evidence to support the assertions in [1] that there is any kind of "instantaneous propagation," because this is relativistically meaningless.

Although both the Bell locality condition and FIN concern forms of locality, it is prima facie difficult to see how FIN, which limits the speed of transmission of information about a single response, could imply Bell locality, which concerns an independence condition on probabilities, i.e. the frequencies of a sequence of measurements. In fact, the implication is false, for otherwise it would, in particular hold in QM itself (as well as as in any putative relativistic extensions of QM). However, information bits, being truth values of properties, are values of projection operators in QM, and cannot be transmitted faster than the speed of light, although their expectations can change instantaneously. Thus, FIN is true in relativistic QM, even though Bell locality is false.

This disproves the remark in [1] that "Nature is non-local, i.e. FIN is wrong." But let us see how this impression might have arisen.

We agree that the observation of the response of a in our experiment does indeed "instantaneously change the state" of the two particles from the singlet state $|\uparrow>|\downarrow> - |\downarrow>|\uparrow>$ to the new state, say, $|\uparrow>|\downarrow>$. However, this merely means that the probabilities of the outcomes of future interactions have changed, and there are simple instances that make it clear that such "instantaneous" changes of state do not violate relativity.

After all, relativity is not violated by the remark that if I find you here in Princeton NJ, the probability that you are in Sydney NSW instantaneously changes to 0.

Again, B, on Mars, knows instantly that *if* a ball has recently been dropped from a height of 16 feet by A on Earth, *then* it will have hit the ground approximately 1 second later.

In our case, experimenter A's observation of the squared spin for a in a given direction told him that an observation of b in the same direction would find the same answer with probability 1 rather than 1/3 or 2/3. The correct statement, that if both particles have been observed in the same direction the results will agree, is relativistically invariant.

What these examples illustrate is *not* any kind of instantaneous transmission, but rather the fact that information that is already present at each of two spatially separated places does not *need* to be transmitted. Known physical laws (that a person cannot be in two places at once, that g on Earth is 32 ft/sec/sec, and the TWIN correlation) are examples of information that is available anywhere.

Of course the TWIN correlation is more subtle than the other two examples, since it refers to correlation between spins that may only take values in future measurements, but the principle is just the same – every physical law, no matter how subtle, provides information that is accessible all over the universe, and so does not need to be transmitted. In the context of QM, TWIN is just a consequence of the conservation of angular momentum.

Physical theories contain general laws, which have no reference to locations in space or time, as well as contingent facts located in space-time, which are usually called initial conditions. The conservation of angular momentum is a general law and TWIN is a sub-law dealing with the case of two spin 1 particles of total spin 0. Thus, TWIN has no reference to space or time and does not need to be transmitted. A contingent instance of this law is afforded by measurements of both particles in the same direction. The fact that these resulting spins only exist after a future measurement plays no role in this description.

Our insistence upon the relativistically correct way of viewing the EPR experiment is hardly new. Every careful text-book on quantum mechanics stresses the fact that observables of a system take values only upon decoherent interaction with another system, such as a measuring apparatus.

We quote from [3], a well-known text by Bohm, written more than fifty years ago (before he became a Bohmian!), in which he introduced the spin version of EPR:

"Thus, for a given atom, *no* component of the spin of a given variable exists with a precisely defined value, until interaction with a suitable system, such as a measuring apparatus, has taken place. ... Thus, in every instance in which particle No. 1 develops a definite spin component in, for example, the z-direction, the wave function of particle No. 2 will automatically take such a form that it guarantees the development of the opposite value of σ_z if this particle also interacts with an apparatus which measures the same component of the spin."

We also refer the reader to the paper [4] for cogent arguments leading to the same conclusion.

2. LORENTZ INVARIANCE

Bassi and Ghirardi claim that the non-locality of QM permits a GRW theory to be Lorentz invariant only in a stochastic sense.

"Such an extension must be also non-local, if it aims at reproducing quantum correlations for EPR types of experiments, in particular when they are performed at space-like separated regions; in one way or another, the jump process, even though it is triggered locally, must 'propagate' practically instantaneously."

This is because of their claim of an instantaneous response of particle a to the measurement of the spin of particle b in some direction. However, as we have pointed out above this is the result of the reification of the theoretical notions of *state* and *probability.* A state is a concept within QM whose function is to predict the probabilities of future events. That it really is no more than this follows from a well-known theorem of Gleason [5] that the QM state can be recovered from the probability predictions about physical observations that QM derives from it.

Most physicists agree that observables do not take values unless the system undergoes a suitable interaction, such as a measurement of the observable. This is clear when the initial state of the system is not an eigenstate of the observable. Some think it is innocuous to allow observables to already have a value if the state is an eigenstate of the observable, even if the observable has not been measured. This is not correct, and leads to the difficulties of instantaneous responses in the EPR situation.

As we stated in [2]:

> "... if a triple experiment has found $x \to 1$, $y \to 1$, $z \to 0$, we certainly know that $S_x^2 = S_y^2 = 1$, but many physicists would also say that 'we also know that $S_w^2 = 1$ for any other direction w perpendicular to z,' (since the probability predicted for this assertion is 1). More modestly, we would say only that '*if a measurement is made in direction w, it will find $S_w^2 = 1$*'."

Similarly, in the EPR experiment, if b is found to have spin up in the z direction, then we say that a measurement of the spin of a in the same direction will find it down. To say, in these circumstances, that S_w^2 is already 1 or the spin is already down is, in our view, to be guilty of a simple confusion. After all, one does not say that an astronomical event like an eclipse has already happened as soon as it has been predicted with certainty.

The language [1] uses invites such a confusion. Since in their view the state changes instantaneously, and the state affects probabilities of future responses of the particle, [1] regards it as acceptable for their putative GRW theories to have the same property:

> "But now we see that the state of particle a has changed because of something *outside* its backward light-cone (the jumps acting on the device used by B at a spacelike separated distance), and this change will affect the response of a to spin-measurements. Moreover, the new state of particle a depends on the choice of directions chosen by B to measure the square of the spin of particle b."

However, the description we have given of QM's prediction for our spin experiment – that if and when both measurements are made they will yield the same answer – is fully relativistically invariant, unlike the above putative GRW description of the same phenomena. Our deduction from the Free Will Theorem was that there can be no mechanism for reduction (such as the GRW proposals) that is as fully invariant as the QM predictions.

After describing the QM predictions for our spin experiment in an invariant way, we went on to say that:

> "Those who would say more might not make any mistaken predictions, but their opinions about what happens are not consistent with relativity theory, unlike our more modest ones."

Our final quotation from [1] illustrates this:

> "... the outcome of the measurement that A performs on *a does* – indeed it *must* – depend on the outcome of the measurement performed by B on b (or vice-versa), in particular, on the choice of directions made by B ... ."

We find this confession quite astonishing, because it can be seen as grossly violating two different principles.

On the one hand, Bassi and Ghirardi profess to accept the experimenters' free will. But it is a curious kind of freedom they grant experimenter B, who is constrained to make only those choices of directions that are compatible with particle a's responses, which will occur only 5 minutes later in some inertial frame. (B could have complete freedom of choice only if the responses of a to the 33 directions that A might choose were to form a 101-function, which the Lemma of Section 2 of [2] proves impossible.)

On the other hand, it is an equally gross violation of causality for a's response to be conditioned by B's decisions, which in another inertial frame will be taken only 5 minutes later.

3. INFORMATION

We defined the "information" mentioned in FIN to consist of *bits*, each of which codes the truth value of some property of the universe. The issue that this is "vague" is raised in [1]. We did not give a precise definition of properties for a good reason. Only a final theory of the universe will tell us what the ultimate properties of the world are. For QM they correspond to projection operators. In an extension of QM such as GRW or a hidden variable theory such as Bohm's the set of properties may well change. We have purposely left the notion of properties and the corresponding information bits as general as possible in order that the Free Will Theorem be as strong as possible.

Bassi and Ghirardi say that Bell's locality condition

$$p_\lambda^{\mathrm{A}B}(x, y; n, m) = p_\lambda^{\mathrm{A}}(x; n, \star)\, p_\lambda^{\mathrm{B}}(y; \star, m)\,.$$

is expressed in clearer terms than FIN. However, that condition has a parameter λ, corresponding to our information bits, that takes values in a set which he leaves equally undefined and vague for the same reason that our properties are undefined; that is in order to prove as strong a theorem as possible.

Nevertheless, Bell's locality condition is a precise statement about probabilities involving the parameter λ, just as our functional hypothesis and FIN are precise conditions about information.

We deliberately avoided probabilities in [2] for three reasons. First, because the contradiction that proves our theorem is about individual observations, not about probabilities associated with sequences of observations. It follows that [1]'s discussion of stochastic conditions is irrelevant. Second, because probabilities are theoretical entities that are not Lorentz invariant and do not satisfy FIN. In Section 2, we saw how an uncritical use of these ideas has led to the impression in [1] that the predictions of quantum mechanics are relativistically invariant only in some weak sense. Third, the notion of probability is problematic not only in QM but even in classical physics, where the naive frequency definition has been justly criticized. There are also different opinions as to whether these probabilities are ontological (about the real world), or merely epistemological (only about our knowledge of the world).

In any case, Bassi and Ghirardi's problems with the precise definition of information are beside the point. Their aim is to exclude jumps from information:

> "... we think that the jump processes should not be regarded as information...they cannot be known ahead of time, they cannot be controlled, and they cannot be used to convey other information..."

In this way they allow the jumps to be instantaneously propagated. However, whether jumps count as information is in fact irrelevant, since they convey to particle a the macroscopic information about B's choice of direction and b's response. It does not matter that the jumps, being stochastic, cannot be subsequently used as signals, since they have already transmitted macroscopic information.

4. RESPONSE TO TUMULKA

We finish by briefly discussing Tumulka's recent paper [6]. He claims to have found three flaws in the argument in [2]. The first claim repeats the mistake in [1] of identifying our FIN with Bell's locality condition. We have already dealt with this in Section 1: FIN is true while Bell locality is false. We find it surprising that Bassi, Ghirardi and Tumulka think we wish "to save locality" despite the non-local correlations of our axiom TWIN and its consequence, of which we were fully aware, that Bell locality is false.

The supposed second and third flaws are more technical. What is important for us is the way Tumulka's discussion of each of them makes clear that, along with Bassi and Ghirardi, he allows particle a's response to depend on the supposedly free choices made by experimenter B.

We quote from his discussion of the supposed second flaw:

> "Freedom is true in rGRWf in the sense that the theory provides, for any given external fields, a distribution of flashes."

The external fields here are those freely chosen by the experimenters, while the flashes determine the particles' responses, so this quotation implies that a's response depends on B's choices.

In his claim of a third flaw, Tumulka quotes our deduction:

> "Now we defined α' to be independent of x, y, z, but it is also independent of w, since there are coordinate frames in which B's experiment happens later than A's."

He objects that this is not a consequence of FIN, in view of [2]'s technical definition of "effective transmission of information". However, in relativity contexts it is clearly a consequence of causality, and by apparently denying it, Tumulka shows once again that he allows particle a's response to be affected by the supposedly free decisions that B will take at a later time in experimenter A's frame.

So Tumulka's discussion shows that he permits the same peculiarity that Bassi and Ghirardi have candidly admitted to, and that we reject as a gross violation of both causality and the Free Will Assumption.

Tumulka also criticizes us for not making a detailed examination of his theory, rGRWf, which he claims is an explicit counter-example to our assertion that a relativistic GRW theory is impossible. We did not need to since rGRWf, having no provision for interactions, is plainly not a counter-example.

He states that this does not matter because "interaction plays no essential role for EPR experiments." It is true that the two particles are not in interaction once they are space-like separated. However, rGRWf is meant to be a theory that accounts for reduction resulting from measurement. The non-local correlations of EPR are manifested only upon measurements of the spins, say by interaction with a magnetic field and a screen in a Stern-Gerlach experiment. Without these measurements there is nothing to "explain". The instantaneous propagation of the jumps of GRW or the flashes of rGRWf are meant to transmit this reduction resulting from the measurement interaction between the particles. A theory like rGRWf that has no interactions cannot be claimed to deal with the EPR experiment. All earlier attempts to construct a relativistic GRW theory have foundered on precisely this problem of including interactions (see [7]). The Free Will Theorem shows that this is inevitable for a fully relativistic theory.

References

[1] A. Bassi, G. C. Ghirardi, arXiv: quant-ph/0610209 (1 Dec 2006).

[2] J. Conway, S. Kochen, Found. Phys. **36** (10), 1441 (2006).

[3] D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951).

[4] M. O. Scully, Y. Aharanov, B.-G. Englert, in *Mysteries, Puzzles, and Paradoxes*, ed. R. Bonifacio, (Am. Inst. Phys., Woodbury 1999).

[5] A. M. Gleason, J. Math. and Mech., **6**, 885 (1957).

[6] R. Tumulka, arXiv: quant-ph/0611283-v2 (6 Dec 2006).

[7] A. Bassi, G. C. Ghirardi, Phys. Rep. **379** (5-6), 257 (2003).